Exercise 13

Evaluate the limit and justify each step by indicating the appropriate properties of limits.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Multiply the fractions together.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \lim_{x\to\infty} \frac{(2x^2 - 7)\frac{1}{x^2}}{(5x^2 + x - 3)\frac{1}{x^2}}$$

Use the distributive property.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \lim_{x\to\infty} \frac{2 - \frac{7}{x^2}}{5 + \frac{1}{x} - \frac{3}{x^2}}$$

The limit of a quotient is the quotient of the limits.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \frac{\lim\limits_{x\to\infty} \left(2 - \frac{7}{x^2}\right)}{\lim\limits_{x\to\infty} \left(5 + \frac{1}{x} - \frac{3}{x^2}\right)}$$

The limit of a sum (difference) is the sum (difference) of the limits.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \frac{\lim\limits_{x\to\infty} 2 - \lim\limits_{x\to\infty} \frac{7}{x^2}}{\lim\limits_{x\to\infty} 5 + \lim\limits_{x\to\infty} \frac{1}{x} - \lim\limits_{x\to\infty} \frac{3}{x^2}}$$

Evaluate all the limits.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \frac{2 - 0}{5 + 0 - 0}$$

Simplify the result.

$$\lim_{x\to\infty} \frac{2x^2 - 7}{5x^2 + x - 3} = \frac{2}{5}$$